The LNM Institute of Information Technology, Jaipur

Written Test for M.Sc. Mathematics Admissions 2019

Name: ______________________ Application Id: ______________________

Instructions

1. The examination is of 2 hours duration. There are a total of 43 questions carrying 50 marks. All questions are compulsory.
2. Questions 1-22 each has four choices out of which only one choice is the correct answer. Q.1-Q.22 carry 1 mark each. Wrong answer will result in **NEGATIVE** marks. For Q.1-Q.22, 1/3 marks will be deducted for each wrong answer.
3. In Questions 23-29, there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Q. 23-29 carry 2 marks each. For Q. 23-29, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions.
4. For Questions 30-43, the answer needs to be written in the space provided. No choices will be shown for these type of questions. Questions Q.30-Q.43 carry 1 mark each. There is **NO NEGATIVE** marking for Q.30-Q.43.
5. Charts, graph sheets, tables, calculator, cellular phone or electronic gadgets are NOT allowed in the examination hall.

Notations

1. $\mathbb{N}$ set of all natural numbers $1, 2, 3, \cdots$
2. $\mathbb{Q}$ set of all rational numbers
3. $\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$, similar is the meaning of $\mathbb{R}^3, \mathbb{R}^n$.
4. $\mathbb{C}$ is set of all complex numbers and $\mathbb{C}^2 = \{(z_1, z_2) : z_1, z_2 \in \mathbb{C}\}$.
5. f_x partial derivative of f with respect to x and f_y denotes partial derivative of f with respect to y
6. $\hat{i}, \hat{j}, \hat{k}$ unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively

1. For the differentiability of a real-valued function of two real variables f at a point (x_0, y_0), the continuity of f_x and f_y at (x_0, y_0) is

   (a) necessary and sufficient

   (b) necessary but not sufficient

   (c) not necessary but sufficient

   (d) neither necessary nor sufficient

2. $\lim\limits_{x\to\infty} x \sin\left(\frac{1}{x}\right) =$

   (a) 1

   (b) 0

   (c) ∞

   (d) none of the above

3. Which of the following set is closed subset of $\mathbb{R}$?

   (a) $\mathbb{Q}$

   (b) $\mathbb{N}$

   (c) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

   (d) $\mathbb{Q} \cap [0, 1]$

4. If $f(x, y, z) = x^2y + y^2z + z^2x$ for all $(x, y, z) \in \mathbb{R}^3$ and

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k},$$

   then the value of $\nabla\cdot(\nabla\times\nabla f)+\nabla\cdot(\nabla f)$ at $(1, 1, 1)$ is

   (a) 0

   (b) 3

   (c) 6

   (d) 9

5. Classify the following differential equation

$$e^x \frac{dy}{dx} + 3y = x^2 y$$

   (a) Separable and not linear

   (b) Linear and not separable

   (c) Both separable and linear

   (d) Neither separable nor linear

6. Let $(G, \cdot)$ be a commutative group of order $7!$. Then G has a subgroup of order

   (a) 11

   (b) 35

   (c) 13

   (d) None of these

7. The basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $\alpha_1 = (1, 1, 2, 4), \alpha_2 = (2, 1, 5, 2), \alpha_3 = (1, 1, 4, 0), \alpha_4 = (2, 1, 1, 6)$ is

   (a) $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$

   (b) $\{\alpha_1, \alpha_2, \alpha_3\}$

   (c) $\{\alpha_1, \alpha_2\}$

   (d) $\{\alpha_1\}$

8. Consider the function $f(x, y) = \dfrac{x^2y + xy^2}{|x| + |y|}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = \lim\limits_{(x,y)\to(0,0)} f(x, y)$, if it exists. Then at the point $(0, 0)$,

   (a) f is not defined

   (b) f is not continuous

   (c) f is continuous but not differentiable

   (d) f is differentiable

9. The maximum magnitude of the directional derivative for the surface $x^2 + xy + yz = 9$ at the point $(1, 2, 3)$ is along the direction
   (a) $\mathbf{i} + \mathbf{j} + \mathbf{k}$
   (b) $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
   (c) $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
   (d) $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

10. Consider the differential equation $(2y^2 + 3x)dx + 2xydy = 0$, which is not exact. Then by finding an appropriate integrating factor, the solution of this differential equation is given by
    (a) $x^2y^2 + x^3 = C$
    (b) $x^2 + y^2 = C^2$
    (c) $x^2(y^2 + 1) = C$
    (d) $x^2 + y^3 = C$

11. Let $y = C_1y_1 + C_2y_2$ is a general solution of the differential equation $y'' + 6y' + 9y = 0$. Which of the following option is correct?
    (a) As $x \to \infty$, $y \to C_1$ for any value of C_2
    (b) The behavior of y as $x \to \infty$ depends on the values of C_1, C_2.
    (c) As $x \to \infty$, $y \to 0$ for any value of C_1, C_2
    (d) As $x \to \infty$, $y \to \infty$ for any value of C_1, C_2

12. Consider the "triangular region" in the first quadrant bounded on the left by the $y-$axis and on the right by the curves $y = \sin x$ and $y = \cos x$. Then the volume of the solid S obtained by revolving this region about the $x - axis$ is
    (a) 1
    (b) -1
    (c) $\frac{\pi}{2}$
    (d) $\frac{\pi}{3}$

13. If C is a smooth curve in $\mathbb{R}^3$ from $(-1, 0, 1)$ to $(1, 1, -1)$, then value of the line integral $\int_C (2xy + z^2)dx + (x^2 + z)dy + (y + 2xz)dz$ is
    (a) 0
    (b) 1
    (c) 2
    (d) 3

14. How many continuous functions $f : \mathbb{R} \to \mathbb{R}$ can be defined such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$.
    (a) Infinitely many
    (b) 2
    (c) 4
    (d) None of above

15. Let $f : (0, 1] \to \mathbb{R}$ be a continuous function.
    (a) $f((0, 1])$ is a compact subset of $\mathbb{R}$.
    (b) $f((0, 1])$ is a connected subset of $\mathbb{R}$.
    (c) $f((0, 1])$ is a bounded subset of $\mathbb{R}$.
    (d) f is strictly increasing on $\mathbb{R}$.

16. Which of the following statement is true?
    (a) Every finite group of prime order is not cyclic.
    (b) Every group of order 6 is abelian.
    (c) Every subgroup of an abelian group is normal.
    (d) The centre of a group is not normal subgroup of the group.

17. Let $(G, \cdot)$ be a group. The equality $(a \cdot b)^n = a^n \cdot b^n$ is true if n is

(a) only positive integer.

(b) only negative integer.

(c) only zero integer.

(d) an integer.

18. The series

$$\sum_{n=1}^{\infty} \frac{p(p-1)\cdots(p-n+1)}{n!} x^n$$

(a) diverges for $|x| < 1$

(b) converges for $|x| > 1$

(c) diverges for $|x| < 2$

(d) none of the above

19. The value of $\frac{d^{10}}{dx^{10}}(x^2-1)^{10}$ at $x = 1$ is

(a) $2^{10} \cdot 10!$

(b) $3^{10} \cdot 10!$

(c) $4^{10} \cdot 10!$

(d) none of the above

20. If $a_{n+1} = \sqrt{2 + a_n}$, for each $n \in \mathbb{N}$ and $a_1 = \sqrt{2}$, then the sequence $(a_n)_{n \geq 1}$ converges to

(a) -1

(b) 2

(c) $\frac{1+\sqrt{5}}{2}$

(d) $\frac{1-\sqrt{5}}{2}$

21. Which of the following set of vectors $\alpha = (a_1, a_2, \cdots, a_n) \in \mathbb{R}^n$ is a subspace of $\mathbb{R}^n, (n > 3)$?

(a) all α such that $a_2 \geq 0$

(b) all α such that $a_1 + 2a_2 = a_3$

(c) all α such that $a_1 a_3 = 0$

(d) all α such that $a_2 = a_3^4$

22. Let V be the vector space of all 2×2 matrices over the field F. Let W_1 be the set of matrices of the form $\begin{pmatrix} x & -x \\ y & z \end{pmatrix}$ and W_2 is set of matrices of the form $\begin{pmatrix} a & b \\ -a & c \end{pmatrix}$. Then dimensions of $W_1, W_2, W_1 + W_2, W_1 \cap W_2$ are, respectively

(a) 3, 3, 3, 2

(b) 3, 3, 3, 3

(c) 3, 3, 4, 1

(d) 3, 3, 4, 2

23. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = x^6 - 2x^2y - x^4y + 2y^2.$$

Which of the following statements is/are TRUE?

(a) f is continuous but not differentiable at origin.

(b) f has all the directional derivatives at origin.

(c) f has a saddle point at origin.

(d) f is not continuous at $(0, 0)$.

24. Which of the following maps T from $\mathbb{R}^2$ into $\mathbb{R}^2$ are linear transformations?

(a) $T(x_1, x_2) = (x_2, x_1)$

(b) $T(x_1, x_2) = (\sin x_1, x_2)$

(c) $T(x_1, x_2) = (x_1 + 2, x_2)$

(d) $T(x_1, x_2) = (x_1 - x_2, 0)$

25. The series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

(a) converges for $|x| < \frac{1}{3}$

(b) converges for $x = -\frac{1}{3}$

(c) converges for $x = \frac{1}{3}$

(d) converges for $|x| \leq \frac{1}{3}$

26. Let $(G, \cdot)$ be a group of order n. It is abelian, if n is

(a) 5

(b) 6

(c) 15.

(d) 7

27. Which of the following statement is correct?

(a) Every compact subset of $\mathbb{R}$ is complete.

(b) Every complete subspace of $\mathbb{R}$ is compact.

(c) Every bounded subset of $\mathbb{R}$ is compact.

(d) Every complete subset of $\mathbb{R}$ is closed.

28. If $\vec{F}(x, y, z) = x\ \hat{i} + y\ \hat{j} + z\ \hat{k}$ for $(x, y, z) \in \mathbb{R}^3$, then which of the following are true?

(a) $\nabla \times \vec{F} = \vec{0}$.

(b) $\oint_C \vec{F}.d\vec{r} = 0$, where C is any closed curve.

(c) There exists a scalar function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\vec{F} = \nabla\phi$.

(d) $\nabla \cdot \vec{F} = 0$

29. Consider the initial value problem

$$\frac{dy}{dx} = y^{2/3},\ y(0) = 0.$$

Then following function(s) satisfy the above IVP

(a) 0

(b) x^3

(c) $(x/3)^3$

(d) None of these

30. Consider the function $f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Then

$$f_x(0, 0) = \cdots\cdots$$
$$f_y(0, 0) = \cdots\cdots$$

Ans. $f_x(0, 0) = 0, f_y(0, 0) = 0$

31. Let T be the linear operator on $\mathbb{C}^2$ defined by $T(x_1, x_2) = (x_1, 0)$. Let B be the standard ordered basis for $\mathbb{C}^2$ and let $B' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i), \alpha_2 = (-i, 2)$, then the matrix representation of T relative to the pair B, B' is

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Ans. $T = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix}$

32. If $s_n = n$ for every $n \in \mathbb{N}$, then as $n \to \infty$ the sequence $\sigma_n := \dfrac{s_1 + s_2 + \cdots + s_n}{n}$ tends to

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

Ans. ∞

33. The converse of Lagrange theorem is true if group is

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

Ans. Finite commutative group.

34. The Taylor series of $\sin^2(x)$ is

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$

Ans. $\dfrac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{(2x)^{2n}}{2n!}$.

35. Let $\phi(x, y, z) = x^2 - y + z^2$ for $(x, y, z) \in \mathbb{R}^3$. Then the directional derivative of ϕ at the point $(1, 2, 1)$ in the direction of $4\hat{i} - 2\hat{j} + 4\hat{k}$ is

....

Ans. 3

36. The graphs of members of the one-parameter family $x^3 + y^3 = 3cxy$ are called Folia of Descartes. This family is an implicit solution of the following first-order differential equation

....

Ans: $\dfrac{dy}{dx} = \dfrac{y(y^3 - 2x^3)}{x(2y^3 - x^3)}$.

37. The maximum value of $f(x, y) = (x - 1)^2 + (y - 2)^2$ on the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 45\}$ is equal to

....

Ans. 80

38. If one solution of $xy'' - (2x + 1)y' + (x + 1)y = 0$ is $y_1 = e^x$, then second solution y_2 is given by

....

Ans. $x^2 e^x$

39. The function f, whose tangent has slope $x^3 - \dfrac{2}{x^2} + 2$ for each value of x and whose graph passes through the point $(1, 3)$ is

....

Ans: $\dfrac{x^4}{4} + \dfrac{2}{x} + 2x - \dfrac{5}{4}$

40. The interval of convergence if the series $\displaystyle\sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n}$ is

....

Ans. $[\frac{1}{3}, 3)$.

41. Let $(G, \cdot)$ be a group, H and K finite subgroups of G such that $o(H)$ and $o(K)$ are relatively prime. Then $o(H \cap K)$

....

Ans. 1.

42. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) := \dfrac{1}{x^4 - 2x^2 + 7}$. Then f has local minima at

$x =$ ....

Ans. $1, -1$

43. Let F be a subfield of the field of complex numbers and let T be the function from F^3 into F^3 defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$, then rank and nullity of T are

....

Ans. The rank of T is 2 and The nullity of T is 1.

Answer Key for Exam $\boxed{\text{A}}$

1. For the differentiability of a real-valued function of two real variables f at a point (x_0, y_0), the continuity of f_x and f_y at (x_0, y_0) is

   (a) necessary and sufficient

   (b) necessary but not sufficient

   $\boxed{\text{(c)}}$ not necessary but sufficient

   (d) neither necessary nor sufficient

2. $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) =$

   $\boxed{\text{(a)}}$ 1

   (b) 0

   (c) ∞

   (d) none of the above

3. Which of the following set is closed subset of $\mathbb{R}$?

   (a) $\mathbb{Q}$

   $\boxed{\text{(b)}}$ $\mathbb{N}$

   (c) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

   (d) $\mathbb{Q} \cap [0, 1]$

4. If $f(x, y, z) = x^2y + y^2z + z^2x$ for all $(x, y, z) \in \mathbb{R}^3$ and

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k},$$

   then the value of $\nabla \cdot (\nabla \times \nabla f) + \nabla \cdot (\nabla f)$ at $(1, 1, 1)$ is

   (a) 0

   (b) 3

   $\boxed{\text{(c)}}$ 6

   (d) 9

5. Classify the following differential equation

$$e^x \frac{dy}{dx} + 3y = x^2y$$

   (a) Separable and not linear

   (b) Linear and not separable

   $\boxed{\text{(c)}}$ Both separable and linear

   (d) Neither separable nor linear

6. Let $(G, \cdot)$ be a commutative group of order $7!$. Then G has a subgroup of order
   (a) 11
   $\boxed{\text{(b)}}$ 35
   (c) 13
   (d) None of these

7. The basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $\alpha_1 = (1,1,2,4), \alpha_2 = (2,1,5,2), \alpha_3 = (1,1,4,0), \alpha_4 = (2,1,1,6)$ is
   (a) $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$
   (b) $\{\alpha_1, \alpha_2, \alpha_3\}$
   $\boxed{\text{(c)}}$ $\{\alpha_1, \alpha_2\}$
   (d) $\{\alpha_1\}$

8. Consider the function $f(x,y) = \dfrac{x^2y + xy^2}{|x| + |y|}$ if $(x,y) \neq (0,0)$ and $f(0,0) = \lim\limits_{(x,y)\to(0,0)} f(x,y)$, if it exists. Then at the point $(0,0)$,
   (a) f is not defined
   (b) f is not continuous
   (c) f is continuous but not differentiable
   $\boxed{\text{(d)}}$ f is differentiable

9. The maximum magnitude of the directional derivative for the surface $x^2 + xy + yz = 9$ at the point $(1,2,3)$ is along the direction
   (a) $\mathbf{i} + \mathbf{j} + \mathbf{k}$
   $\boxed{\text{(b)}}$ $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
   (c) $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
   (d) $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

10. Consider the differential equation $(2y^2 + 3x)dx + 2xydy = 0$, which is not exact. Then by finding an appropriate integrating factor, the solution of this differential equation is given by
    $\boxed{\text{(a)}}$ $x^2y^2 + x^3 = C$
    (b) $x^2 + y^2 = C^2$
    (c) $x^2(y^2 + 1) = C$
    (d) $x^2 + y^3 = C$

11. Let $y = C_1y_1 + C_2y_2$ is a general solution of the differential equation $y'' + 6y' + 9y = 0$. Which of the following option is correct?
    $\boxed{\text{(a)}}$ As $x \to \infty$, $y \to C_1$ for any value of C_2
    (b) The behavior of y as $x \to \infty$ depends on the values of C_1, C_2.
    (c) As $x \to \infty$, $y \to 0$ for any value of C_1, C_2
    (d) As $x \to \infty$, $y \to \infty$ for any value of C_1, C_2

12. Consider the "triangular region" in the first quadrant bounded on the left by the $y-$axis and on the right by the curves $y = \sin x$ and $y = \cos x$. Then the volume of the solid S obtained by revolving this region about the $x - axis$ is

(a) 1

(b) -1

$\boxed{\text{(c)}}$ $\dfrac{\pi}{2}$

(d) $\dfrac{\pi}{3}$

13. If C is a smooth curve in $\mathbb{R}^3$ from $(-1, 0, 1)$ to $(1, 1, -1)$, then value of the line integral $\displaystyle\int_C (2xy + z^2)dx + (x^2 + z)dy + (y + 2xz)dz$ is

(a) 0

(b) 1

$\boxed{\text{(c)}}$ 2

(d) 3

14. How many continuous functions $f : \mathbb{R} \to \mathbb{R}$ can be defined such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$.

(a) Infinitely many

(b) 2

$\boxed{\text{(c)}}$ 4

(d) None of above

15. Let $f : (0, 1] \to \mathbb{R}$ be a continuous function.

(a) $f((0, 1])$ is a compact subset of $\mathbb{R}$.

$\boxed{\text{(b)}}$ $f((0, 1])$ is a connected subset of $\mathbb{R}$.

(c) $f((0, 1])$ is a bounded subset of $\mathbb{R}$.

(d) f is strictly increasing on $\mathbb{R}$.

16. Which of the following statement is true?

(a) Every finite group of prime order is not cyclic.

(b) Every group of order 6 is abelian.

$\boxed{\text{(c)}}$ Every subgroup of an abelian group is normal.

(d) The centre of a group is not normal subgroup of the group.

17. Let $(G, \cdot)$ be a group. The equality $(a \cdot b)^n = a^n \cdot b^n$ is true if n is

(a) only positive integer.

(b) only negative integer.

(c) only zero integer.

$\boxed{\text{(d)}}$ an integer.

18. The series

$$\sum_{n=1}^{\infty} \frac{p(p-1)\cdots(p-n+1)}{n!} x^n$$

(a) diverges for $|x| < 1$

(b) converges for $|x| > 1$

(c) diverges for $|x| < 2$

$\boxed{\text{(d)}}$ none of the above

19. The value of $\frac{d^{10}}{dx^{10}}(x^2-1)^{10}$ at $x=1$ is

$\boxed{\text{(a)}}$ $2^{10} \cdot 10!$

(b) $3^{10} \cdot 10!$

(c) $4^{10} \cdot 10!$

(d) none of the above

20. If $a_{n+1} = \sqrt{2+a_n}$, for each $n \in \mathbb{N}$ and $a_1 = \sqrt{2}$, then the sequence $(a_n)_{n \geq 1}$ converges to

(a) -1

$\boxed{\text{(b)}}$ 2

(c) $\frac{1+\sqrt{5}}{2}$

(d) $\frac{1-\sqrt{5}}{2}$

21. Which of the following set of vectors $\alpha = (a_1, a_2, \cdots, a_n) \in \mathbb{R}^n$ is a subspace of $\mathbb{R}^n$, $(n > 3)$?

(a) all α such that $a_2 \geq 0$

$\boxed{\text{(b)}}$ all α such that $a_1 + 2a_2 = a_3$

(c) all α such that $a_1 a_3 = 0$

(d) all α such that $a_2 = a_3^4$

22. Let V be the vector space of all 2×2 matrices over the field F. Let W_1 be the set of matrices of the form $\begin{pmatrix} x & -x \\ y & z \end{pmatrix}$ and W_2 is set of matrices of the form $\begin{pmatrix} a & b \\ -a & c \end{pmatrix}$. Then dimensions of $W_1, W_2, W_1 + W_2, W_1 \cap W_2$ are, respectively

(a) 3, 3, 3, 2

(b) 3, 3, 3, 3

(c) 3, 3, 4, 1

$\boxed{\text{(d)}}$ 3, 3, 4, 2

23. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = x^6 - 2x^2y - x^4y + 2y^2.$$

Which of the following statements is/are TRUE?

(a) f is continuous but not differentiable at origin.

$\boxed{\text{(b)}}$ f has all the directional derivatives at origin.

$\boxed{\text{(c)}}$ f has a saddle point at origin.

(d) f is not continuous at $(0, 0)$.

24. Which of the following maps T from $\mathbb{R}^2$ into $\mathbb{R}^2$ are linear transformations?

$\boxed{\text{(a)}}$ $T(x_1, x_2) = (x_2, x_1)$

(b) $T(x_1, x_2) = (\sin x_1, x_2)$

(c) $T(x_1, x_2) = (x_1 + 2, x_2)$

$\boxed{\text{(d)}}$ $T(x_1, x_2) = (x_1 - x_2, 0)$

25. The series $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

$\boxed{\text{(a)}}$ converges for $|x| < \frac{1}{3}$

(b) converges for $x = -\frac{1}{3}$

$\boxed{\text{(c)}}$ converges for $x = \frac{1}{3}$

(d) converges for $|x| \leq \frac{1}{3}$

26. Let $(G, \cdot)$ be a group of order n. It is abelian, if n is

$\boxed{\text{(a)}}$ 5

(b) 6

$\boxed{\text{(c)}}$ 15.

$\boxed{\text{(d)}}$ 7

27. Which of the following statement is correct?

$\boxed{\text{(a)}}$ Every compact subset of $\mathbb{R}$ is complete.

(b) Every complete subspace of $\mathbb{R}$ is compact.

(c) Every bounded subset of $\mathbb{R}$ is compact.

$\boxed{\text{(d)}}$ Every complete subset of $\mathbb{R}$ is closed.

28. If $\vec{F}(x, y, z) = x\ \hat{i} + y\ \hat{j} + z\ \hat{k}$ for $(x, y, z) \in \mathbb{R}^3$, then which of the following are true?

$\boxed{\text{(a)}}$ $\nabla \times \vec{F} = \vec{0}$.

$\boxed{\text{(b)}}$ $\oint_C \vec{F}.d\vec{r} = 0$, where C is any closed curve.

$\boxed{\text{(c)}}$ There exists a scalar function $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $\vec{F} = \nabla\phi$.

(d) $\nabla \cdot \vec{F} = 0$

29. Consider the initial value problem

$$\frac{dy}{dx} = y^{2/3}, \ y(0) = 0.$$

Then following function(s) satisfy the above IVP

$\boxed{\text{(a)}}$ 0

(b) x^3

$\boxed{\text{(c)}}$ $(x/3)^3$

(d) None of these

30. Consider the function $f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$ Then

$$f_x(0,0) = \cdots\cdots$$
$$f_y(0,0) = \cdots\cdots$$

Ans. $f_x(0,0) = 0, f_y(0,0) = 0$

31. Let T be the linear operator on $\mathbb{C}^2$ defined by $T(x_1, x_2) = (x_1, 0)$. Let B be the standard ordered basis for $\mathbb{C}^2$ and let $B' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i), \alpha_2 = (-i, 2)$, then the matrix representation of T relative to the pair B, B' is

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

Ans. $T = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix}$

32. If $s_n = n$ for every $n \in \mathbb{N}$, then as $n \to \infty$ the sequence $\sigma_n := \dfrac{s_1 + s_2 + \cdots + s_n}{n}$ tends to

....

Ans. ∞

33. The converse of Lagrange theorem is true if group is

....

Ans. Finite commutative group.

34. The Taylor series of $\sin^2(x)$ is

....

Ans. $\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2x)^{2n}}{2n!}$.

35. Let $\phi(x, y, z) = x^2 - y + z^2$ for $(x, y, z) \in \mathbb{R}^3$. Then the directional derivative of ϕ at the point $(1, 2, 1)$ in the direction of $4\hat{i} - 2\hat{j} + 4\hat{k}$ is

....

Ans. 3

36. The graphs of members of the one-parameter family $x^3 + y^3 = 3cxy$ are called Folia of Descartes. This family is an implicit solution of the following first-order differential equation

....

Ans: $\frac{dy}{dx} = \frac{y(y^3 - 2x^3)}{x(2y^3 - x^3)}$.

37. The maximum value of $f(x, y) = (x-1)^2+(y-2)^2$ on the set $\{(x, y) \in \mathbb{R}^2 : x^2+y^2 \leq 45\}$ is equal to

....

Ans. 80

38. If one solution of $xy'' - (2x + 1)y' + (x + 1)y = 0$ is $y_1 = e^x$, then second solution y_2 is given by

....

Ans. x^2e^x

39. The function f, whose tangent has slope $x^3 - \frac{2}{x^2} + 2$ for each value of x and whose graph passes through the point $(1, 3)$ is

....

Ans: $\frac{x^4}{4} + \frac{2}{x} + 2x - \frac{5}{4}$

40. The interval of convergence if the series $\sum_{n=1}^{\infty}\frac{(3x-2)^n}{n}$ is

....

Ans. $[\frac{1}{3}, 3)$.

41. Let $(G, \cdot)$ be a group, H and K finite subgroups of G such that $o(H)$ and $o(K)$ are relatively prime. Then $o(H \cap K)$

....

Ans. 1.

42. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) := \dfrac{1}{x^4 - 2x^2 + 7}$. Then f has local minima at

$x = \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$ Ans. $1, -1$

43. Let F be a subfield of the field of complex numbers and let T be the function from F^3 into F^3 defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$, then rank and nullity of T are

. . . .

Ans. The rank of T is 2 and The nullity of T is 1.